Modeling Opinion Dynamics in a Social Network

Qi Gu, Eugene Santos Jr.
Thayer School of Engineering
Dartmouth College, Hanover, NH, USA
{qi.gu, eugene.santos.jr }@dartmouth.edu

Eunice E. Santos
National Center for Border Security and Immigration
The University of Texas at El Paso, El Paso, TX, USA
eesantos@utep.edu

***Abstract*—Opinion dynamics is a complex procedure that entails a cognitive process when dealing with how a person integrates influential opinions to form a revised opinion. In this work, we present a new approach to model opinion dynamics by treating the opinion on an issue as a product inferred from one's knowledge bases, where the knowledge bases keep growing and updating through social interaction. A general impact metric is proposed to evaluate the likelihood of a person adopting the opinions from others. Specifically, a set of domain-independent influential factors is selected based on social and communication theories, but the weights of these factors are missing. Though the opinions from different actors are not integrated linearly like traditional methods, we show that the factor weights can be efficiently learned via regression. We validated the effectiveness of our model by comparing against a baseline model on both synthetic and real datasets. The contribution of this paper lies with 1) a novel opinion dynamics model that emphasize the dependencies between knowledge pieces; 2) proof that the classical DeGroot model is a special case of our model under certain conditions; and, 3) to the best of our knowledge, this is the first work to try and uncover the mechanism that guides the selection of opinions in the real world by modeling opinion change.**

***Keywords*—opinion dynamics; opinion formation; knowledge fusion; influential factors;**

I. INTRODUCTION

The process of interpersonal influence that affects individuals' beliefs and opinions lies at the heart of socialization, identity and decision-making. It also serves as a guide to how people change their behaviors by taking into account the attitudes and opinions of others with whom they interact [1]. Evidence of the reliance on others' opinions in one's decision can be found in many fields, from advertising with social cues [26] to the diffusion of political views [2]. Yet, despite its importance in various forms of social policies, to fully understand opinion dynamics through social processes is never a trivial job. Firstly, not everyone holds the same base of knowledge. It is often the case that when information is hard to process or access due to incompleteness, one would shape their opinions by interacting with others who hold views on the given issues [17]. Moreover, even if two people do not exchange views on a specific problem, due to the causal relationships between knowledge fragments, the newly acquired information from the conversation could influence their opinions toward that problem. For example, a person may decide not to buy a Japanese car after being told by her friend that a political issues between Japan and her country have become severely strained, even though their conversation had nothing to do with car-buying. In addition, the art of understanding personal opinions rests not only on the measurement of people's views but also on understanding the motivations behind those views. Therefore, it becomes increasingly important to model opinion dynamics by considering diversity in individual knowledge bases, the dependencies between knowledge fragments, and the intentions behind the opinion change.

On the other hand, a person will neither fully adopt nor completely disregard the opinions of other people, but takes into account the opinions of others to a certain extent in forming her own opinion. However, how an individual adopts the beliefs of others may vary in different situations. For instance, people are inclined to take the opinions from their close friends more seriously than from random people. Trust-based models have been well developed to measure the magnitude of trust between people or to provide recommendations in social networks [3][4][5]. However, social trust depends on a host of factors that can never be easily modeled computationally [3]. Moreover, even if a person trusts someone's words, he may not want to change her opinions. In our work, we focus on how people view and adopt the opinions held by the people they have communicated with. Though how the knowledge bases are expanded and updated through social interaction seems complex, there may exist a core mechanism guiding the selection of opinions such that we can use it to control or predict future opinion dynamics. To uncover such mechanism, we start by considering several domain independent influential factors that have been shown to be highly related to opinion formation by communication scholars and social theories [19][10][21]. But, we show that our model is not restricted to these factors. Newly discovered factors can be easily incorporated into the model.

Bayesian approaches have been widely used to represent opinions due to its ability at capturing causal relationships between variables [6][7]. For example, Garg et al. [6] introduces a Bayesian network (BN) [12] based divergence minimization framework to integrate opinions from different sources in order to solve the problem of standard opinion pooling. However, people's knowledge-based system, is necessarily associated with some degree of incompleteness, which turns out to be problematical to BNs, as they require a completely specified conditional

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probability table (CPT). BNs also require that information be topologically ordered which further restricts their general applicability to real-world situations. To overcome these limitations, we apply a probabilistic framework called Bayesian Knowledge Bases (BKBs) [9] to represent individual opinions due to its flexibility for modeling incomplete information and allowance of cycles at the variable level. BKBs have been extensively used to model complex intent-driven scenarios [7].

In our previous work, a Finite Fusion Model (FFM) was proposed for detecting and tracking hidden sources in a time-variant scenario given a sequence of belief distributions encoded as BKBs [11]. In this paper, we apply FFM to opinion dynamics modeling by incorporating a general impact metric. Specifically, a person forms her new opinion by aggregating the knowledge fragments absorbed from the people she interacted within a given period, where the impact of each communication is determined by the impact metric. We design the metric by combining the influential factors in a linear fashion such that the weight for each factor can be learned from data. We show that though opinions from different people are not linearly fused, the relationship between the opinion change and influential factors can be transformed into a regression problem and be solved efficiently. To demonstrate the effectiveness of our approach, we compare against a baseline model on both synthetic data and real data. A thorough analysis of the situations in which our approach becomes equivalent to classic opinion formation models is provided as well.

II. Background and Related Works

A. Related Works

Opinion formation is a key process to understand and explain opinion dynamics. Works based on social influence network theory have made remarkable progress in showing how networks of interpersonal influence contribute to the formation of interpersonal consensus in complex circumstances [10][1][14]. One of the classical ways is to form people's revised opinions through a weighted averaging of the influential opinions, where the weights between two actors on the network can be either static or time-variant [1][13][18]. However, most of these works are theoretical models where the strong assumptions attached to these methods, such as fixed social structure or simultaneity restrict them to be only applicable to extremely simple scenarios. Other approaches take into account some factors while leaving out others. For example, Tessone et al. [16] studies the effect of population size in opinion spread, but only focuses on consensus situations. Wu & Huberman [17] predicts the evolution of a set of opinions by considering the structure of the social network. However, they assume that the influences from each neighbor are the same, which is hardly true in the real world. In this work, the role of social network structure is not our focus. Many approaches to opinion formation consider single opinion dynamics but overlook the connections between the knowledge fragments [15][16][17][18]. In contrast, we treat the opinion as a product inferred from one's knowledge base, where the knowledge base expands and changes through the communications. By doing so, we increase the flexibility of quantifying an individual's opinions such that more insights on opinion dynamics could be brought out via inferring over the knowledge bases.

In modeling influence in social networks, efforts have been made on characterizing the influence of each individual on information diffusion based on the social network structure and interpersonal interaction [27][24]. However, different from these approaches whose main goals are to target the influential people that can persuade a great number of people in their society, we emphasize on finding the rules of how one adopts another's opinion. In order to model the impact in a person's opinion, we start by selecting a set of correlated factors: 1) Relationship, characterizes the closeness between people, is a strong indicator of how likely one may accept another's opinion. It forms the social environment, e.g. family or friends that people are willing to adjust their attitudes to conform to their group members [19]. However, without frequent contact, even two close friends may differ so much on their knowledge bases that one can hardly convince the other. 2) The second factor is the recent contact frequency. Research from Chong & Druckman [10] shows that frequent communication allows one to deliver her opinions more credibly to the public. 3) Similarity has been used as a metric to evaluate the relationship between people in social network analysis [20]. Instead of measuring the similarity of personal characteristics, such as age and location, we measure the opinion-wise similarity, as the former one can be highly correlated to the relationship. Sniderman and Theriault [21] finds that individuals favored the idea that was consistent with their own values.

B. Bayesian Knowledge-base

In this work, we apply BKBs [9] to model a person's opinion at each time period. BKBs are a rule-based probabilistic model that represents possible world states and their (causal) relationships using a directed graph. BKBs are an alternative to BNs by specifying dependence at the instantiation level (versus BNs that specify only at the random variable level); by allowing for cycles between variables; and, by loosening the requirements for specifying complete probability distribution. BKBs collect the conditional probability rules (CPR) in an "if-then" style. Each instantiation of a random variable is represented by an I-node and the rule specifying the conditional probability of an I-node is encoded in an S-node with a certain weight/probability. (Fig. 1 (right) presents an example BKB representing a fragment of one's knowledge bases.) The opinion value on a given issue can be reasoned from the BKB. For example, the marginal probability of "vote for the demographic" quantifies one's attitude on supporting the demographic.

C. Finite fusion model

Santos et al. [11] proposes a Finite Fusion Model (FFM) to model the revised knowledge base influenced by hidden sources. In the context of FFM, the knowledge bases of each source are represented by a BKB. People revise their knowledge bases by fusing the knowledge fragments from others using the BKB fusion algorithm [22]. The idea of the algorithm is to take the union of all input fragments by incorporating source nodes, indicating the source and reliability of the fragments. Fig. 1 shows an example of the knowledge fusion, where two BKB fragments from actor 1 and actor 2 are fused into one. A useful property of knowledge fusion is that it considers the impacts from multiple sources when constructing explanations for any evidence observed. Thus, the opinions/beliefs inferred from the fused BKB are not a linear combination of the original ones. Take Fig. 1 as an example, let K_1, K_2 be the knowledge base for actor 1 and 2, and let K′ be the fused BKB. Then the probability of "Vote" in K' becomes

$$\begin{aligned} p'(Vote = Y) &= [r_1 * p_1(IIP = Y) + r_2 * p_2(IIP = Y)] \\ &* [(r_3 * p_1(Vote = Y|IIP = Y) + r_4 \\ &* p_2(Vote = Y|IIP = Y)] \\ &= (r_1 0.3 + r_2 0.6)(r_3 0.9 + r_4 0.7) \end{aligned}$$

$p'(Vote = Y)$ does not equal the weighted average, i.e., $r_3 p_1(Vote = Y)+r_4 p_2(Vote = Y)$ in most cases (some special cases will be discussed in a later section).

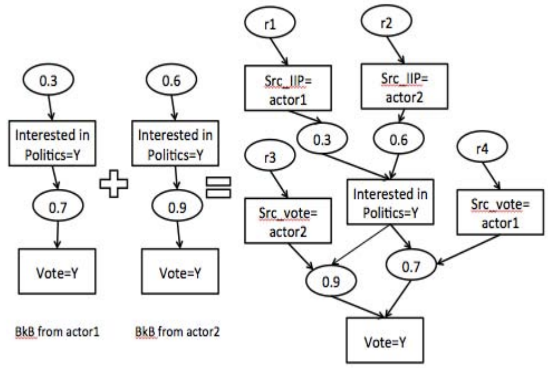


Figure 1: Example of knowledge fusion with square blocks and ovals representing I-nodes and S-nodes, respectively

III. MODELS

Without an explicit mathematical formulation, it is hard to analyze and understand the process of opinion dynamics.

A. Opinion Formation Model

People do not like/dislike a thing for no reason. We treat all opinions as the reasoning results from ones' knowledge base. We assume that all knowledge fragments that serve as input to our model are valid BKBs. The example in Fig. 1 shows that one's integrated opinion is a high-order polynomial function with the reliability of each knowledge fragment being indeterminate. For example, $p'(Vote = Y)$ is a second order polynomial of r_1, r_2, r_3 and r_4.

From FFM, the individual knowledge base at time t $(t = [1:T])$ can be viewed as an integration of the previous one and the knowledge fragments acquired through the interaction. We assume that everyone's initial BKBs have the same structure, but different distribution. The trick is that if one's BKB misses one part of the information, we can always treat the corresponding conditional probabilities as 0 (The transformed BKBs are still valid). Given an actor 0 whose initial knowledge base is K_0^0 (K_t^i denotes the knowledge base for actor i at time step t), let p_t^0 represents the distribution of the BKB after fusing knowledge pieces from the people he contacted from $t-1$ to t, i.e. K_t^0=fusion$(K_{t-1}^0, K_{t-1}^1, \ldots K_{t-1}^n)$. Then, the opinion on a specific issue v, represented by the probability can be calculated from Bayes theorem:

$$p_t^0(v) = \sum_j p_t^0\left(v \middle| pa^j(v)\right) * p_t^0(pa^j(v)) \qquad (1)$$

where $pa^j(v)$ denotes the jth parent set of v and $p_t^0\left(v \middle| pa^j(v)\right)$ is simply the weight of the jth S-node q_j pointing to v in K_t^0. Let $w_t(q)$ be the weight of the S-node q. Then,

$$p_t^0(v) = \sum_j w_t(q_j) * p_t^0(pa^j(v)) \qquad (2)$$

$$w_t(q_j) = \frac{p_t^0(v, pa^j(v))}{p_t^0(pa^j(v))}$$

For each S-node q_j, $pa^j(v)$ consists of all precedent I-nodes of q_j. So $pa^j(v)$ also contains the source node src_j indicating where a particular piece of knowledge fragment comes from. Let $r_t(src_j)$ be the reliability of such a knowledge fragment and let $\overline{pa}^j(v) = pa^j(v) \backslash src_j$. Since the sources nodes are independent of any random variables in a fused BKB, we have

$$p_t^0\left(pa^j(v)\right) = r_t(src_j) * p_t^0\left(\overline{pa}^j(v)\right) \qquad (3)$$

After combining equation (2) and (3), we have

$$p_t^0(v) = \sum_j w_t(q_j) * r_t(src_j) * p_t^0(\overline{pa}^j(v)) \qquad (4)$$

Let r_t^i $(i = [0:n])$ be the reliabilities for K_{t-1}^i $(i = [0:n])$, then from [22], equation (4) can be rewritten into

$$p_t^0(v) = \sum_i^n r_{ti} \sum_{q_j \in \sigma_i} w_{t-1}(q_j) * p_t^0(\overline{pa}^j(v)) \qquad (5)$$

where σ_i is the set of S-nodes pointing to v in K_{t-1}^i. Let $\rho_t^i = \sum_{q_j \in \sigma_i} w_{t-1}(q_j) * p_t^0(\overline{pa}^j(v))$, then

$$p_t^0(v) = \sum_i^n r_t^i * \rho_t^i \qquad (6)$$

where

$$\sum_{\mathrm{i}}^{\mathrm{n}} r_t^i = 1$$

In fact, ρ_t^i represents each actor's view on v given the distribution of v's the precedent variables being updated from p_{t-1}^0 to p_t^0.

B. General Impact metric

In the previous sections, we discussed the importance of each of the three factors: friendship, recent contact frequency and opinion similarity that may influence how one takes other's opinions. Next, we propose a linear impact metric that combines these factors in a coherent scheme. Linear models have been widely used in trust-based approaches due to its simplicity and intuitiveness [23][26]. At time t, let $X_t(i,j)$ denote the closeness between actor i and j from actor i's view, scaled from 0 to 1, $Y_t(i,j)$ denotes the normalized contact frequency, i.e. number of contacts with j divided by the total number of contacts with all others from time $t-1$ to t, $Z_t(i,j)$ denotes the opinion difference on the target issue, i.e. $|p_{t-1}^i(v) - p_{t-1}^j(v)|$. Then, the likelihood of actor i adopting the opinions from actor j, represented by $r_t(i,j)$ can be defined as:

$$r_t(i,j) = \alpha X_t(i,j) + \beta Y_t(i,j) + \gamma Z_t(i,j) + C$$

where α, β, γ and C are unknown weights/coefficients that need to be learned. We replace r_t^i with the upper formula. Then, equation (6) turns into

$$p_t^0(v) = \sum\nolimits_{i=0}^{n} [\alpha X_t(0,i) + \beta Y_t(0,i) + \gamma Z_t(0,i) + C] * \rho_t^i$$

$$= \left(1 - \sum\nolimits_{i=1}^{n} [\alpha X_t(0,i) + \beta Y_t(0,i) + \gamma Z_t(0,i) + C]\right) * \rho_t^0 + \sum\nolimits_{i=1}^{n} [\alpha X_t(0,i) + \beta Y_t(0,i) + \gamma Z_t(0,i) + C] * \rho_t^i$$

Rearranging the terms, we get

$$\begin{aligned} p_t^0(v) - \rho_t^0 &= \alpha \sum\nolimits_{i=1} X_t(0,i)(\rho_t^i - \rho_t^0) \\ &+ \beta \sum\nolimits_{i=1} Y_t(0,i)(\rho_t^i - \rho_t^0) \\ &+ \gamma \sum\nolimits_{i=1} Z_t(0,i)(\rho_t^i - \rho_t^0) \\ &+ C \sum\nolimits_{i=1} (\rho_t^i - \rho_t^0) \end{aligned} \tag{7}$$

C. Parameter Estimation

Let n be the number of people in the group under consideration. To model the opinion dynamics, we look into the opinions over a number of time periods. Let $P = \{p_0^i, p_1^i, p_2^i, \dots, p_T^i\}$ ($i = [1{:}n]$)) be a sequence of belief distributions generated over T time periods. Then, given the friendship network X and the contact network Y over time, the goal is to learn the factor weights α, β, γ and C from opinion change. Note, considering that the measure is taken from each actor's view, neither the friendship network nor the contact network needs to be symmetric.

As shown in equation (5), an actor's opinion $p_t^0(v)$ depends on the distribution of v's parent variables at time t. So what if $p_t^0(v)$ represents the prior probabilities of the variables with no precedent variables? From the fusion algorithm, the fusion of the prior probabilities is simply the weighted average of each input prior, i.e.

$$p_t^0(v) = \sum\nolimits_{i}^{n} r_t^i * p_{t-1}^i(v)$$

Similar to how we derive equation (7), we have

$$\begin{aligned} p_t^0(v) - p_{t-1}^0(v) &= \alpha \sum\nolimits_{i=1} X_t(0,i)(p_{t-1}^i(v) - p_{t-1}^0(v)) \\ &+ \beta \sum\nolimits_{i=1} Y_t(0,i)(p_{t-1}^i(v) - p_{t-1}^0(v)) \\ &+ \gamma \sum\nolimits_{i=1} Z_t(0,i)(p_{t-1}^i(v) - p_{t-1}^0(v)) \\ &+ C \sum\nolimits_{i=1} (p_{t-1}^i(v) - p_{t-1}^0(v)) \end{aligned} \tag{8}$$

So far, we have shown how to model the opinion change as a function of influential factors using equation (7) and (8) for one particular issue/I-node v. We apply the strategy to all possible I-nodes. The solution of α, β, γ and C can be found via regression.

The complete algorithm can be described more formally as follows:

Algorithm 1.
$[\alpha, \beta, \gamma, C]$ = Influential-factor-weights (P, X, Y)
1. A ← ∅; U ← ∅;
2. k ← 1
3. **for** every instantiation/state v ; $t = 1{:}T$; actor $i = 1{:}n$
4. **if** v has no precedent variable
5. $U(k) \leftarrow p_t^i(v) - p_{t-1}^i(v)$
6. $d_{ij} \leftarrow p_{t-1}^j(v) - p_{t-1}^i(v)$ $(j = 1{:}n, j \neq i)$
7. **else**
8. $U(k) \leftarrow p_t^i(v) - \rho_t^i$
9. $d_{ij} \leftarrow \rho_t^j - \rho_t^i$ $(j = 1{:}n, j \neq i)$
10. **end if**
11. $A(k,:) \leftarrow [\sum_j X_t(i,j) * d_{ij}\ ,\ \sum_j Y_t(i,j) * d_{ij},$
12. $\sum_j |p_{t-1}^i(v) - p_{t-1}^j(v)| * d_{ij}\ ,\ \sum_j d_{ij}]$
13. k ← k + 1
14. **end for**
15. $[\alpha, \beta, \gamma, C]$ = regression(U, A)

IV. Experiments

In what follows, we present results of experiments that were carried out on both simulated data and a real world dataset. We start by introducing the baseline we compared with in the experiments.

A. Baseline:

DeGroot model is one of the classical opinion formation model, in which one forms her opinions by taking an average over opinions acquired through communication [18]. Specifically, they represent the opinion on a given issue by a real number. At time $t + 1$, person i adjusts her opinion on issue v by taking a weighted mean with weight $r_t(i, j)$ for the opinion of person j at time t. So if we represent the opinion value by the probability she supports for that issue, then the process can be formed as:

$$p_t^i(v) = \sum_j^n r_t(i, j) * p_{t-1}^j(v),$$

where

$$\sum_j^n r_t(i, j) = 1.$$

Other variations of this model includes the Friedkin-Johnsen model [1], which assumes that a person would adhere to her initial opinion $p_0^i(v)$ to a certain extent. Though there are some other models in which the weight matrix may vary depending on time or opinion itself [15], their opinion formation process is still linear.

To compare with our work, we assign the reliability to each person using the general impact metric introduced in the last section. Then, the influential factor weights can be transformed into a regression in a similar way to equation (8). In fact, if there are no dependencies between different variables/issues, then our approach will devolve into the classical linear model.

B. Synthetic data

1) Experimental Setup

To evaluate the effectiveness of our method, we first randomly generate an n-by-n friendship network X. Then the contact networks $Y = \{Y_1, Y_2, \ldots, Y_T\}$ for every time step are simulated based on the friendship network but with a certain deviation. The assumption here is that close people tend to be in contact more frequently. We also simulate each individual's initial opinion distribution $\{p_0^1, p_0^2, p_0^3, \ldots, p_0^n\}$ from n predefined BKBs. At time t $(t = 1{:}T)$, we fuse one's original BKB $K_{t-1}^i$with the BKB fragments from the people he contacted, where the contacting information is gained from the Y_t. The reliability for each fragment is calculated using the general impact metric, where the factor weights are predefined for simulation, e.g. $[\alpha, \beta, \gamma, C] = [0.4, 0.2, -0.2, 0.2]$.

2) Performance Evaluation

In the first experiment, we test the performance of our approach on learning the influential factors through opinion dynamic modeling. Since the problem can be solved via regression, we use RSE (Relative Squared Error) to measure the error rate of the regression model. RSE works well when the errors of models are measured in different units. Let $\mathrm{u_i}, \mathrm{u'_i}$ and $\bar{\mathrm{u}}_\mathrm{i}$ be the real opinion change (simulated opinion values), the predicted change and mean of the real values, then RSE can be defined as:

$$\mathrm{RSE} = \sum_i (u'_i - u_i)^2 \,/\, \sum_i (\bar{u}_i - u_i)^2$$

Also, to evaluate the prediction ability, we apply a 5-fold cross-validation on the regression.

We run multiple experiments to test our performance in terms of different social network size, initial knowledge bases and network connection degree. In particular, we first increase the number of nodes in the friendship network and contact network while maintaining other settings the same. We run each experiment for 10 times. Fig. 2(a) plots the average RSE using our approach and the baseline respectively. Then, we vary the complexity of the predefined BKBs from which we generate the initial opinion distribution by increasing the number of the variables. By doing so, we test whether our method is capable of handling complex knowledge bases. The results are shown in Fig. 2(b). Last but not least, we measure the network connection degree as a summarized degree centrality, i.e. $C_D = \frac{\sum_{i=1}^n deg(i)}{(n^2-n)/2}$, where $deg(i)$ calculates the number of links that node i has. If the contact network is fully connected, then C_D equals to 1. We depict the regression error in terms of C_D in Fig. 2(c).

From Figs. 2(a), (b) and (c) we can see that the error of using our method is consistently smaller than the baseline. In addition, when there are more variables in the BKB, our advantages appear to be more significant (shown in Fig. 2(b)). The main reason is that a complex knowledge system is always accompanied by strong dependencies between knowledge fragments. The baseline model, however, fails to capture such dependencies, as it assumes that people's opinions on different issues are independent of each other. Moreover, as shown in Fig. 2(c), the error decreases as the connection degree gets closer to 1. Actually, a higher connection degree will lower the impact from each of the neighbor. So the errors from the inaccurate factor weights learning will be averaged out after fusing in a large portion of neighbors' opinions.

Another way to evaluate our learning performance is to directly measure the similarity between the factor weights learned from two methods against the true predefined weights. As the factor weights are represented by a vector, we apply cosine similarity to measure the accuracy, i.e. $\mathrm{sim} = \frac{\mathrm{v_1 \cdot v_2}}{||\mathrm{v_1}||||\mathrm{v_2}||}$. Table 1 reports the average cosine similarity to show our learning capability. As we can see, our approach outperforms the baseline model, which gives us confidence on using our results as a guide to control or predict future opinion dynamics.

TABLE I. AVERAGE SIMILARITY WITH THE TRUE FACTOR WEIGHTS

	Baseline	Proposed
Avg Cos Sim	0.5867	0.7438

3) Convergence Analysis

As we discussed earlier, if there is no dependency between different issues or knowledge pieces, then the way we transform the problem will become the same as the way we deal with the baseline. However, it is not true in the real world, especially when the knowledge bases evolve over time. To better understand the connections between our method and the baseline and to answer whether our method is compatible with the classical model, we examine under what situation will our method produce similar results as the baseline.

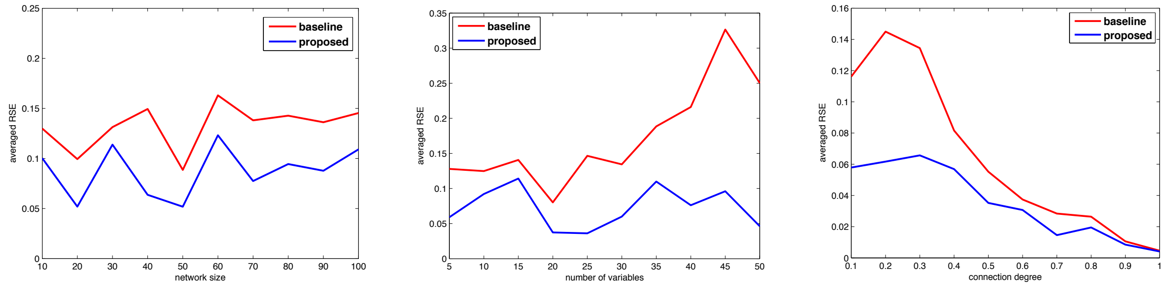


Figure 2. Comparison of two methods in terms of (a) network size; (b) the complexity of the initial knowledge bases; (c) conneciton degree C_D.

Lemma 1, if everyone's initial BKBs share the same conditional probabilities, i.e. $p_0^i(v|pa(v)) = p_0^j(v|pa(v))$, then our approach becomes equivalent to the baseline.

Proof, from Rosen et al. [8], for actor i, the probability on a given issue v can be calculated as the summation of the probabilities of all possible inferences where v is true. So at time t, we have:

$$p_t^i(v) = \sum_{\tau \in I_v} p_t^i(\tau) = \sum_{\tau \in I_v} \prod_{q \in \tau} w_t^i(q)$$

where I_v represents the set of inferences containing v, $w(q)$ is the weight of the S-node q. Then, the aggregated opinion at time $t+1$ using baseline method will be:

$$p_{t+1}^i(v)_{baseline} = \sum_{j=1}^{n} r_j * p_t^j(v) = \sum_{j=1}^{n} r_j \sum_{\tau \in I_v^j} \prod_{q \in \tau} w_t^j(q)$$

Considering that our opinion formation method is based on BKB fusion, for every I-node in an inference, the weight of its supporting S-node in the fused BKB can be viewed as the average over the weights of the corresponding S-nodes in the input BKB fragments, i.e. $w'^i_t(q) = \sum_{j=1}^n r_j * w_t^j(q)$. So using our method, $p_{t+1}^i(v)$ becomes:

$$p_{t+1}^i(v)_{proposed} = \sum_{\tau \in I_v} \prod_{q \in \tau} \sum_{j=1}^n r_j * w_t^j(q)$$

Given the condition that $w_t^i(q) = w_t^j(q)$, we can now derive:

$$p_{t+1}^i(v)_{baseline} = \sum_{\tau \in I_v^j} \prod_{q \in \tau} w_t^j(q) = p_{t+1}^i(v)_{proposed} \blacksquare$$

To illustrate Lemma 1, we run an experiment by decreasing the deviations between the conditional probabilities in each BKB. Fig. 3 shows the mean error difference of using two methods. As we can see, the differences between two methods shrink when the distributions of different initial BKBs get closer.

So, what can we gain from lemma 1? In fact, in a closed social environment, if there is no new information coming in, then both of our method and the baseline will lead to opinion convergence, where the convergence rate highly depends on the social connection degree. From Lemma 1, if people's opinions are close to each other, e.g. converge to certain values, then the advantage of our methods will diminish. (Note, our performance will never be worse than the baseline.) We test the convergence rate by measuring the normalized square of interpersonal opinion difference at each time step t, i.e. $OD_t = \sum_{i,j} \sum_v (p_t^i(v) - p_t^j(v))^2 / n^2 V$, where V is the total number of states/issues in the BKB. We assume that everyone's initial BKBs have different distribution. Then, to examine how the social connection degree affects the convergence rate, we choose three different connection degrees: $C_D = 0.2$,0.5 and 0.8 and the results are plotted in Figs. 4. As shown in the figure, the convergence rate increases with the connection degree. This finding also explains why in Fig. 2 (c), the performance difference decreases when C_D gets larger. Additionally, the opinions converge to different values when the degree is low, e.g. 0.3. The fact is that different opinion groups are formed when people only communicate with the people in a subgroup. Moreover, on closer examination of the convergence trend, we see that the opinions formed using our method converge slightly faster than the baseline. The reason is that we do not form opinions linearly but considers the relationships between variables. So the distribution characteristics of higher-level variables could be propagated to the lower-level variables, which speeds up the convergence.

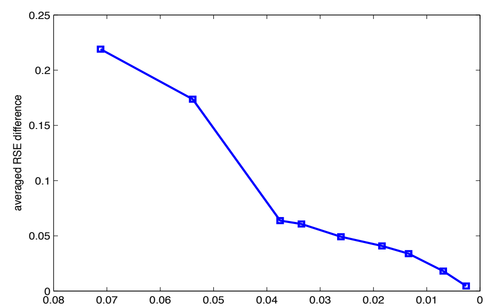
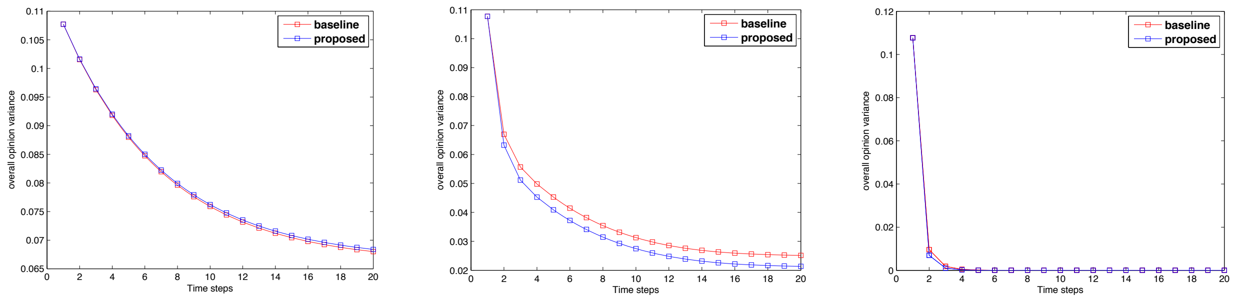


Figure 3. Mean RSE difference between our method and the baseline, i.e. diff=proposed_rse-baseline_rse

In reality though, changes in people's opinions are gradual [25]. Plus, people keep absorbing new information from external sources and keep making new friends. So, it is very unlikely at any point that people hold the same opinions, which makes the conditions described in Lemma 1 hardly to satisfy. This finding is valuable. It makes our approach more applicable to real-world problems.



(a) (b) (c)

Figure 4. Convergence rate comparision between two methods given different connection degree (from left to right: $C_D = 0.2, 0.5$ and 0.8)

C. Real data

To better understand the opinion dynamics in the real world, it is crucial to test our approach on a real dataset. In particular, we conduct our experiment on the adoption of political opinions through face-to-face communication. People's prospects on political issues, especially the voting behaviors can be influenced by various information sources, e.g. media and social interaction. Madan et al. studied how opinions about political candidates and parties, and voting behavior, spread through a social network using mobile sensing data [2]. The study was conducted among approximately 70 undergraduate students living in the same dormitory. They built up a mobile sensing platform to collect the communication data varies from Bluetooth signal transmission to location. The political opinions were captured using three monthly surveys before and after the presidential election in 2008. More descriptions about the data can be found in [2]. To model opinion changes on different issues, we select four questions from the survey. The questions and the possible responses are listed in Tab. 2. Then, we use Bayes Net Toolbox [1] to capture the causal dependencies between variables. The learned causal structure is {Interest_in_politics −> Likely_to_vote; Preferred_party −> who_to_vote} (as partly shown in Fig. 1). As we expected, the candidate that a person decides to vote for is highly dependent on her preferred party. Next, we convert each of the questions into a binary state variable. For each student, her knowledge base is built based on the learned causal structure. Then, the probabilities/opinions for every variable/issue at each time step can be estimated from her survey answers. For example, if one is very interested in politics, then her probability for "interested in politics" is assigned to 0.9. Also, if one decides to vote for McCain, then her probability for "vote for Obama" will be very low.

Now that we have everyone's opinion distribution over time, the next step is to generate the friendship network X and contact network Y. Aside from the communication data and political view data, [2] also provides a survey of the relationship that a subject indicates she has with another, e.g. close friends or political discussant. We quantify the friendship using a single value between 0 (stranger) and 1 (extremely close). Next, we model the face-to-face contact in a period between two students as a function of physical proximity counts and the detected interactions.

TABLE II. SELECTED POLITICAL SURVEY QUESTIONS AND POSSIBLE ANSWERS

Question	Possible Answers
Interested in politics	[Very, Somewhat, Not-at-all]
Preferred political party	[Republican, Independent, Democrat]
Likelihood to vote	[Not-vote, Not-sure, Vote]
Who to vote	[Probably John McCain, Definitely John McCain, Undecided, Probably Barack Obama, Definitely Obama]

We test our approach on the real data and compare the results with the baseline. The first column in Tab. 4 shows the average error from cross-validation. As we can see, the error produced from our method is much lower than the baseline. Moreover, we analyze the learned factor weights between two methods. As plotted in Tab. 3, the value of γ from our method suggests a negative effect of the opinion difference factor, which fits well with our expectation that people are more willing to take opinions that are similar to theirs. In contrast, the baseline model produces a positive coefficient for opinion difference, but negative weights for friendship and the contact frequency, which fails to agree with the social theory findings. Besides, in this particular case, the influence of the friendship appears higher than the other two factors. This indicates two things. 1) For the undergraduate students, their political views are more likely to be affected by their close friends. 2) Their opinions could be influenced by other communication means other than face-to-face, e.g. online chat.

There can be tons of factors that determine how an individual takes others' opinions. In this paper, we only consider three factors that we believe are playing an important role in opinion dynamics. However, we show that our method is not restricted to those three factors. Tab. 4, columns 2 to 4 show the results of using only two factors out of three. It is not hard to see that our method outperforms baseline in all factor combinations. Thus, even

[1] BNT is a toolbox for learning Bayesian network's parameters and structure. Here we only use BNT to learn the causal structure. https://code.google.com/p/bnt/

when new factors are available, our method can be easily extended to accommodate new influential factors.

TABLE III. FACTOR WEIGHTS LEARNED FROM TWO METHODS

	α	β	γ	C
Proposed	0.0768	0.0134	-0.0381	0.1234
Baseline	-0.1389	-0.0001	0.0142	0.1138

TABLE IV. AVERAGE ERROR USING DIFFERENT FACTOR SETS (F = FRIENDSHIP, C= CONTACT, OD = OPINION DIFFERENCE)

	[F,C,OD]	[F,OD]	[F,C]	[C,OD]
Proposed	0.0872	0.0893	0.0928	0.1063
Baseline	0.6467	0.6532	0.6203	0.7509

CONCLUSION

In this work, we proposed a new approach to model opinion dynamics, where the individual opinions are treated as a product inferred from one's knowledge bases. Though how the knowledge bases are expanded and updated through social interaction seems complex, there may exist a core mechanism that guides people to adopt others' opinions or to absorb new information. We aim to uncover the rules behind opinion dynamics by starting with several influential factors that could affect the selection of opinions. Then, we show how to formulate the problem using a mathematical model and solve the factor weights via regression. We conduct multiple experiments on synthetic data. The results validate the effectiveness of our approach over the baseline. A thorough analysis and discussion is provided on the compatibility of our approach. Another main contribution of this work is that we test our approach on modeling the political opinion change using a real world dataset. The error produced by our approach is much smaller than the baseline. Besides that, the factor weights learned from our approach are also consistent with social theory.

In future work, more influential factors should be considered, where non-critical factors tend to have weak weights. However, incorporating more factors could generate lower errors, but cause overfitting as well. So, new factors should be added with a carefully selected penalty term.

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